GATE 2027 EC, Section 7

Communications

Random processes and noise, AM and FM, superheterodyne receivers, information theory, PCM and digital modulation, error probability and error-correcting codes.

41 formulas in 8 topics. Equations are numbered section.topic.formula, for example (7.1.1).

7.1 Random processes and noise

Autocorrelation (7.1.1)

$R_X(\tau) = E[X(t)X(t+\tau)]$; power $= R_X(0)$

Wiener–Khinchin (7.1.2)

$S_X(f) = F\{R_X(\tau)\}$; power $= \int S_X(f)df$

White noise (7.1.3)

$S(f) = N_0/2$; $R(\tau) = (N_0/2)\delta(\tau)$

Through an LTI system (7.1.4)

$S_Y(f) = |H(f)|^2 S_X(f)$

Noise in bandwidth B (7.1.5)

Power $= N_0B$ (ideal filter); thermal: $P = kTB$, ${v^2}_{rms} = 4kTRB$

Noise figure (7.1.6)

$F = SNR_{in}/SNR_{out}$; cascade (Friis): $F = F_1 + (F_2 - 1)/G_1 + \ldots$

7.2 Amplitude modulation

AM signal (7.2.1)

$s(t) = A_c[1 + \mu \cos \omega_m t] \cos \omega_{ct}$; $\mu = A_m/A_c$

From envelope (7.2.2)

$\mu = (V_{max} - V_{min})/(V_{max} + V_{min})$

Power (7.2.3)

$P_t = P_c(1 + \mu^2/2)$; efficiency $\eta = \mu^2/(2 + \mu^2)$ ($\leq 33.3\%$)

Bandwidth (7.2.4)

AM and DSB-SC: $2f_m$; SSB: f_m; VSB: slightly above f_m

Multiple tones (7.2.5)

$\mu_{total} = \sqrt{({\mu_1}^2 + {\mu_2}^2 + \ldots)}$

7.3 Angle modulation

FM index (7.3.1)

$\beta = \Delta f/f_m$; $\Delta f = k_f A_m$

Carson's rule (7.3.2)

$BW \approx 2(\Delta f + f_m) = 2(\beta + 1)f_m$

FM spectrum (7.3.3)

Lines at $f_c \pm nf_m$ with amplitudes $A_cJ_n(\beta)$; total power ${A_c}^2/2$ (constant)

Narrowband FM (7.3.4)

$\beta \ll 1$: $BW \approx 2f_m$

Figure of merit (tone) (7.3.5)

DSB-SC, SSB: 1; AM (envelope): $\mu^2/(2 + \mu^2)$; FM: $3\beta^2/2$

7.4 Superheterodyne receiver

Intermediate frequency (7.4.1)

$f_{IF} = f_{LO} - f_{RF}$ (high-side LO)

Image frequency (7.4.2)

$f_{image} = f_{RF} + 2f_{IF}$

Image rejection (7.4.3)

$\alpha = \sqrt{(1 + Q^2\rho^2)}$, $\rho = f_{image}/f_{RF} - f_{RF}/f_{image}$

7.5 Information theory

Information (7.5.1)

$I = \log_2(1/p)$ bits

Entropy (7.5.2)

$H = -\Sigma\, p_i \log_2 p_i$; maximum $\log_2 M$ when equiprobable

Source coding (7.5.3)

Average length $L \geq H$; efficiency $= H/L$; Huffman gives an optimal prefix code

Mutual information (7.5.4)

$I(X; Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$

Binary symmetric channel (7.5.5)

$C = 1 - H(p)$ bits per use

Shannon–Hartley (7.5.6)

$C = B \log_2(1 + S/N)$

7.6 Pulse and digital modulation

PCM (7.6.1)

$L = 2^n$ levels; bit rate $= n \cdot f_s$; minimum bandwidth $n \cdot f_s/2$

PCM SQNR (7.6.2)

$\approx 6.02n + 1.76$ dB (full-scale sine); each extra bit adds ≈ 6 dB

Delta modulation (7.6.3)

No slope overload if $\Delta \cdot f_s \geq \max|dm/dt|$; tone: $\Delta \cdot f_s \geq 2\pi f_m A_m$

DPCM (7.6.4)

Quantise the prediction error, not the sample

Symbols (7.6.5)

Bits per symbol $= \log_2 M$; $R_s = R_b/\log_2 M$

Nyquist ISI criterion (7.6.6)

Minimum bandwidth $R_s/2$; raised cosine: $B = (R_s/2)(1 + \alpha)$

Null-to-null bandwidth (7.6.7)

BPSK $2R_b$; QPSK R_b; BFSK $\approx |f_1 - f_2| + 2R_b$

7.7 Detection and error probability

Matched filter (7.7.1)

$h(t) = s(T - t)$; maximum output SNR $= 2E/N_0$

MAP and ML (7.7.2)

MAP picks max $P(s_i|r)$; ML = MAP when all symbols are equally likely

Q-function (7.7.3)

$Q(x) = (1/\sqrt{(2\pi)}) \int_x^\infty e^{-t^2/2} dt$

Coherent BER (7.7.4)

BPSK, QPSK: $Q(\sqrt{(2E_b/N_0)})$; BFSK: $Q(\sqrt{(E_b/N_0)})$; OOK (average E_b): $Q(\sqrt{(E_b/N_0)})$

Non-coherent BER (7.7.5)

DPSK: $\frac{1}{2}e^{-E_b/N_0}$; non-coherent BFSK: $\frac{1}{2}e^{-E_b/2N_0}$

7.8 Error-correcting codes

Hamming bound for parity bits (7.8.1)

$2^r \geq m + r + 1$ (m data bits, r parity bits); (7, 4) code has $d_{min} = 3$

Detection, correction (7.8.2)

Detect $d_{min} - 1$ errors; correct $\lfloor (d_{min} - 1)/2 \rfloor$

Code rate (7.8.3)

$R = k/n$

CRC (7.8.4)

Append r zeros (r = degree of g(x)); CRC = remainder of $M(x)x^r \div g(x)$; catches all bursts $\leq$ r bits

My notes