GATE 2027 EC, Section 1

Engineering mathematics

Linear algebra, calculus, differential equations, vector analysis, complex analysis, probability and statistics.

53 formulas in 6 topics. Equations are numbered section.topic.formula, for example (1.1.1).

1.1 Linear algebra

Eigenvalues (1.1.1)

$\det(A - \lambda I) = 0$

$\Sigma\lambda$ = trace(A), $\Pi\lambda$ = det(A)

Eigenvalues of related matrices (1.1.2)

$A^{-1} \to 1/\lambda$, $A^k \to \lambda^k$, $kA \to k\lambda$, $A + kI \to \lambda + k$

Same eigenvectors

Rank–nullity (1.1.3)

$\text{rank}(A) + \text{nullity}(A) = n$ (number of columns)

Ax = b solutions (1.1.4)

Consistent iff $\text{rank}(A) = \text{rank}([A|b])$; unique if that rank $= n$, infinite if $< n$

Ax = 0 (1.1.5)

Non-trivial solution iff $\det(A) = 0$ (square A)

2 × 2 inverse (1.1.6)

$[a\ b;\ c\ d]^{-1} = (1/(ad - bc))\ [d\ -b;\ -c\ a]$

Linear independence (1.1.7)

Vectors are independent iff the only solution of $\Sigma c_i v_i = 0$ is all $c_i = 0$

Special matrices (1.1.8)

Symmetric → real eigenvalues; orthogonal $A^T A = I \to |\lambda| = 1$; skew-symmetric → λ purely imaginary or 0

Cayley–Hamilton (1.1.9)

Every square matrix satisfies its own characteristic equation

Triangular matrix (1.1.10)

Eigenvalues are the diagonal entries

1.2 Calculus

Rolle's theorem (1.2.1)

$f(a) = f(b) \Rightarrow f'(c) = 0$ for some $c \in (a, b)$

Lagrange mean value (1.2.2)

$f'(c) = (f(b) - f(a)) / (b - a)$

Taylor series (1.2.3)

$f(x) = \Sigma\, f^{(n)}(a)(x - a)^n / n!$

Standard series (1.2.4)

$e^x = 1 + x + x^2/2! + \ldots$; $\sin x = x - x^3/3! + \ldots$; $\cos x = 1 - x^2/2! + \ldots$; $\ln(1 + x) = x - x^2/2 + x^3/3 - \ldots$ $(|x| < 1)$

L'Hôpital (1.2.5)

$\lim f/g = \lim f'/g'$ for 0/0 or ∞/∞ forms

Maxima/minima (2 variables) (1.2.6)

$f_x = f_y = 0$; $r = f_{xx}$, $s = f_{xy}$, $t = f_{yy}$; $rt - s^2 > 0$: max if $r < 0$, min if $r > 0$; $rt - s^2 < 0$: saddle

Integration by parts (1.2.7)

$\int u\, dv = uv - \int v\, du$

Useful integrals (1.2.8)

$\int_0^\infty e^{-ax}\, dx = 1/a$; $\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$

Gamma function (1.2.9)

$\Gamma(n + 1) = n\Gamma(n) = n!$; $\Gamma(1/2) = \sqrt{\pi}$

Wallis (1.2.10)

$\int_0^{\pi/2} \sin^n x\, dx = [(n-1)(n-3)\ldots]/[n(n-2)\ldots] \times (\pi/2$ if n even, 1 if n odd)

Improper integral test (1.2.11)

$\int_1^\infty dx/x^p$ converges iff $p > 1$; $\int_0^1 dx/x^p$ converges iff $p < 1$

1.3 Differential equations

First-order linear (1.3.1)

$y' + P(x)y = Q(x)$: $IF = e^{\int P\, dx}$, $y \cdot IF = \int Q \cdot IF\, dx + C$

Exact equation (1.3.2)

$M\, dx + N\, dy = 0$ is exact iff $\partial M/\partial y = \partial N/\partial x$

Constant coefficients (1.3.3)

Roots m of auxiliary equation: distinct → $C_1 e^{m_1 x} + C_2 e^{m_2 x}$; repeated → $(C_1 + C_2 x)e^{mx}$; $\alpha \pm j\beta \to e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

Complete solution (1.3.4)

y = complementary function + particular integral

Euler–Cauchy (1.3.5)

$x^2 y'' + axy' + by = 0$: try $y = x^m \to m(m - 1) + am + b = 0$

Variation of parameters (1.3.6)

$y_p = -y_1 \int (y_2 R/W)dx + y_2 \int (y_1 R/W)dx$, $W = y_1 y_2' - y_2 y_1'$

Separation of variables (PDE) (1.3.7)

Assume $u(x, t) = X(x)T(t)$, split into ODEs with a separation constant

1.4 Vector analysis

Directional derivative (1.4.1)

$D_a f = \nabla f \cdot \hat{a}$

Identities (1.4.2)

$\nabla \cdot (\nabla \times F) = 0$; $\nabla \times (\nabla f) = 0$; $\nabla^2 f = \nabla \cdot \nabla f$

Solenoidal / irrotational (1.4.3)

$\nabla \cdot F = 0 \rightarrow$ solenoidal; $\nabla \times F = 0 \rightarrow$ irrotational ($F = \nabla \varphi$)

Gauss divergence theorem (1.4.4)

$\oint_S F \cdot dS = \int_V (\nabla \cdot F)\, dV$

Stokes' theorem (1.4.5)

$\oint_C F \cdot dl = \int_S (\nabla \times F) \cdot dS$

Green's theorem (1.4.6)

$\oint_C (M\, dx + N\, dy) = \iint_R (\partial N/\partial x - \partial M/\partial y)\, dx\, dy$

1.5 Complex analysis

Cauchy–Riemann (1.5.1)

$u_x = v_y$, $u_y = -v_x$ ($f = u + jv$ analytic)

Euler's formula (1.5.2)

$e^{j\theta} = \cos\theta + j \sin\theta$

Cauchy's integral theorem (1.5.3)

$\oint_C f(z)\, dz = 0$ if f is analytic on and inside C

Cauchy's integral formula (1.5.4)

$f(a) = (1/2\pi j) \oint f(z)/(z - a)\, dz$; $f^{(n)}(a) = (n!/2\pi j) \oint f(z)/(z - a)^{n+1}\, dz$

Residue theorem (1.5.5)

$\oint_C f(z)\, dz = 2\pi j \times \Sigma$ (residues inside C)

Residue at a pole (1.5.6)

Simple: $\lim_{z \to a} (z - a) f(z)$; order m: $[1/(m-1)!] \lim d^{m-1}/dz^{m-1} [(z - a)^m f(z)]$

Laurent series (1.5.7)

$f(z) = \Sigma\, a_n (z - a)^n$ (n from $-\infty$ to ∞); residue $= a_{-1}$

1.6 Probability and statistics

Addition rule (1.6.1)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional, Bayes (1.6.2)

$P(A|B) = P(A \cap B)/P(B)$; $P(A|B) = P(B|A)P(A)/P(B)$

Independence (1.6.3)

$P(A \cap B) = P(A)P(B)$

Mean, variance (1.6.4)

$E[X] = \Sigma x\, p(x)$; $\mathrm{Var}(X) = E[X^2] - (E[X])^2$; $\mathrm{Var}(aX + b) = a^2 \mathrm{Var}(X)$

Binomial (1.6.5)

$P(k) = {}^nC_k\, p^k (1 - p)^{n-k}$; mean np, variance $np(1 - p)$

Poisson (1.6.6)

$P(k) = e^{-\lambda} \lambda^k / k!$; mean = variance = λ

Exponential (1.6.7)

$f(x) = \lambda e^{-\lambda x}$; mean $1/\lambda$, variance $1/\lambda^2$; memoryless

Uniform (a, b) (1.6.8)

mean $(a + b)/2$, variance $(b - a)^2/12$

Normal (1.6.9)

$f(x) = (1/\sigma\sqrt{(2\pi)})\, e^{-(x - \mu)^2/2\sigma^2}$; 68–95–99.7% within 1, 2, 3σ

Mode–median–mean (1.6.10)

Mode $\approx$ 3 Median $-$ 2 Mean (moderately skewed data)

Correlation (1.6.11)

$r = \mathrm{Cov}(X, Y)/(\sigma_x \sigma_y)$, $-1 \leq r \leq 1$

Regression of y on x (1.6.12)

$y - \bar{y} = r(\sigma_Y/\sigma_x)(x - \bar{x})$; product of both regression coefficients $= r^2$

My notes